

Time: 3 hrs

Marks: 80

Note :

- 1) Q. No. 01 is compulsory.
- 2) Solve any three from Q. No. 02 to 06.
- 3) Numbers to the right indicate full marks.
- 4) Use of statistical tables is allowed.

Q. 1. Solve.

- a) If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ find the sum and product of Eigen values A. 5
- b) Integrate the function $f(z) = z^2$ from A(0, 0) to B(1, 1) along straight line AB. 5
- c) Find the Z-Transform of $(k) = a^k$, $k < 0$. 5
- d) A transmission channel has a per-digit error probability $p = 0.01$. Calculate the probability of more than 1 error in 10 received digits using Poisson distribution. 5

Q. 2.

- a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. 6
- b) Find the Z-Transform of $\text{Cos}\left(\frac{k\pi}{4} + \alpha\right)$ $k \geq 0$. 6
- c) Use the dual simplex method to solve the LPP
 Min.. $Z = 2X_1 + 2X_2 + 4X_3$
 $2X_1 + 3X_2 + 5X_3 \geq 2$, $3X_1 + X_2 + 7X_3 \leq 3$, $X_1 + 4X_2 + 6X_3 \leq 5$ $X_1, X_2, X_3 \geq 0$ 8

Q. 3.

- a) Evaluate $\int_C \frac{z^2}{(z-1)(z-2)} dz$ Where C is a circle $|z - 1| = 1$. 6
- b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. 6
- c) Solve the LPP by Big -M method
 Maximize $Z = 3X_1 - 2X_2$
 subject to $2X_1 + X_2 \leq 2$, $X_1 + 3 \geq 3$, $X_1, X_2, \geq 0$. 8

Q. 4.

- a) Find inverse Z transform of $F(z) = \frac{1}{(z-1)(z-3)}$ for i) $|z| < 1$, ii) $1 < |z| < 3$. 6
- b) The following data represent the marks obtained by 12 students in two tests, one held before the coaching and the other after the coaching.
 Test I : 55, 60, 65, 75, 49, 25, 18, 30, 35, 51, 61, 72. 6
 Test II : 63, 70, 70, 81, 54, 29, 21, 38, 32, 50, 70, 80.
 Do the data indicate that the coaching was effective in improving the performance of the students?
- c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z-1)(z+2)}$ about $z = 0$ indicating the region of convergence in each case. 8

Q. 5.

- a) Determine all basic solutions to the following problem
 Max. $Z = x_1 - 2x_2 + 4x_3$ 6
 $x_1 + 2x_2 + 3x_3 = 7$, $3x_1 + 4x_2 + 6x_3 = 15$, $x_1, x_2, x_3 \geq 0$.
- b) Using Normal distribution, find the probability of getting 55 heads in the toss of 100 fair coins. 6
- c) Solve the NLPP
 Optimize $Z = 10x_1 + 8x_2 + 6x_3 + 2x_1^2 + x_2^2 + 3x_3^2 - 100$ 8
 Subject to $x_1 + x_2 + x_3 = 20$, $x_1, x_2, x_3 \geq 0$.

Q. 6.

- a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$. 6
- b) Of the 64 off springs of a certain cross between guinea pigs 34 were red, 10 were black and 20 were white. According to the generic model these numbers should be in the ratio 9 : 3 : 4. Use 2- test to check whether the data are consistent with the model. 6
- c) Max. $Z = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$, Subject to $x_1 + x_2 \leq 2$ and $2x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$ by K-T condition. 8
